CHAPTER 3

Mathematics sample tasks

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The mathematics questions in PISA aim at assessing the capacity of students to draw upon their mathematical competencies to meet the challenges of their current and future daily lives. Citizens have to use mathematics in many daily situations, such as when consulting media presenting information on a wide range of subjects in the form of tables, charts and graphs, when reading timetables, when carrying out money transactions and when determining the best buy at the market. To capture this broad conception, PISA uses a concept of *mathematical literacy* that is concerned with the capacity of students to analyse, reason and communicate effectively as they pose, solve and interpret mathematical problems in a variety of situations including quantitative, spacial, probabilistic or other mathematical concepts.

Mathematics was the focus of the PISA 2003 survey, meaning that more time was dedicated to mathematics testing which allowed a more detailed analysis of the results. The 2006 mathematics results are compared to the 2003 benchmarks, as will be the case for results from future surveys. In 2000 and 2006, mathematics was also assessed, but less comprehensively than in 2003. Key assessment characteristics were established for the 2000 survey and underwent minor modifications for the following surveys.

Mathematics is defined in relation to three dimensions: the *content*, the mathematical *processes* and the *situations*. The first dimension, the *content* of mathematics, is defined primarily in terms of "overarching ideas" and only secondarily in relation to curricular strands. Strands such as numbers, algebra and geometry are commonly used in curricula. The overarching ideas used in PISA reflect the orientation towards real-life situations. For the first survey in 2000 two overarching ideas were assessed: *change and growth* and *space and shape*. These two were selected to allow a wide range of curriculum strands to be represented, without giving undue weight to number skills. In the assessments in 2003 and 2006 four overarching ideas were assessed: *quantity*, *space and shape*, *change and relationships* and *uncertainty*. This is in line with the contemporary view of mathematics as the science of patterns in a general sense. The PISA overarching ideas reflect this: patterns in *space and shape*, patterns in *change and relationships*, patterns in *quantity* form central and essential concepts for any description of mathematics, and they form the heart of any curriculum, at any level. But to be literate in mathematics means more. Dealing with uncertainty from a mathematical and scientific perspective is essential. For this reason, elements of probability theory and statistics give rise to the fourth overarching idea: *uncertainty*.

The second dimension is the *process* of mathematics as defined by general mathematical competencies. Questions are organised into three "competency clusters" (reproduction, connections and reflection) defining the type of thinking skill needed. The first cluster – *reproduction* - consists of simple computations or definitions of the type most familiar in conventional mathematics assessments. The second requires *connections* to be made to solve relatively straightforward problems. The third competency cluster – *reflection* – consists of mathematical thinking, generalisation and insight, and requires students to engage in analysis, to identify the mathematical elements in a situation and to pose their own problems. In general, these processes are in ascending order of difficulty, but it does not follow that one must be mastered in order to progress to the other: it is possible for example to engage in mathematical thinking without being good at computations. These competencies are applied as part of the fundamental process of mathematisation that students use to solve real-life problems. Mathematisation can be broken up into five steps:

- Starting with a problem in reality.
- Organising it according to mathematical concepts and identifying the relevant mathematics.
- Gradually trimming away the reality to transform the real-world problem into a mathematical problem that faithfully represents the situation.
- Solving the mathematical problem.
- Making sense of the mathematical solution in terms of the real situation.

The third dimension is the *situation* in which mathematics is used. PISA identifies four situations: personal, educational or occupational, public (related to the local community or society) and scientific. Each question used in a PISA survey falls into one category of each of the three dimensions. Question 10.1 from the unit Carpenter, for example, is part of the connections competency cluster, using content of the overarching idea quantity and set in an occupational situation. As the last two categorisations are generally fairly obvious, they will not be explicitly mentioned for the questions presented here.

To report the results of PISA 2000 a single mathematics scale was used. The average score on this scale is 500 with two-thirds of students scoring between 400 and 600. In 2003, when mathematics was the major domain, separate scales for each of the four content areas were created in addition to the overall mathematics scale. As in 2000, the average on each scale is 500 with two-thirds of students scoring between 400 and 600. In the 2006 survey, a single mathematics scale was used to gauge performance. The results are compared to the benchmark of 500 score points established by PISA 2003. More information on PISA proficiency scales can be found in Annex A.

It is the policy of PISA that students should be allowed to use calculators and other tools as they are normally used in school. However, the test questions are chosen so that the use of calculators is not likely to enhance a student's performance in the assessment. This chapter presents 50 units. The first 26 units were used in the PISA surveys. Units 27 to 50 were used in developing and testing out the surveys. While it was decided not to include these units in the PISA surveys, they are nevertheless illustrative of the kinds of questions asked in PISA. The questions presented in this chapter are all publicly released PISA mathematics questions. Following the section with questions, answers for all questions are given. For units 1 to 26, a comment box includes score points, the percentage of students who answered correctly across OECD countries and the question category. For country results, refer to Annex B. For units 27 to 50, a comment box lists the question category. Because these units were not used in the final PISA surveys the information regarding score points and percentage of students who answered correctly is not known or it not sufficiently reliable to be presented here.

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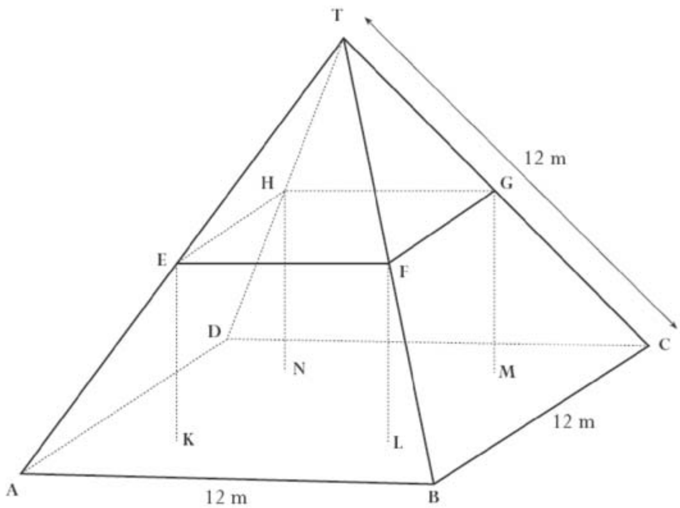
MATHEMATICS UNIT 1: FARMS

Here you see a photograph of a farmhouse with a roof in the shape of a pyramid.

Below is a student's mathematical model of the farmhouse **roof** with measurements added.



The attic floor, ABCD in the model, is a square. The beams that support the roof are the edges of a block (rectangular prism) EFGHKLMN. E is the middle of AT, F is the middle of BT, G is the middle of CT and H is the middle of DT. All the edges of the pyramid in the model have length 12 m.

QUESTION 1.1

Calculate the area of the attic floor ABCD.

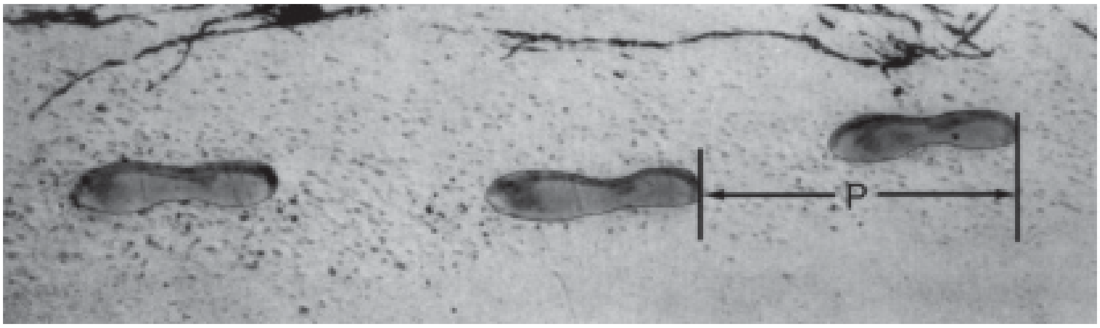
The area of the attic floor ABCD = ________ m^2

QUESTION 1.2

Calculate the length of EF, one of the horizontal edges of the block.

The length of EF = ______________ m

MATHEMATICS UNIT 2: WALKING



The picture shows the footprints of a man walking. The pacelength *P* is the distance between the rear of two consecutive footprints.

For men, the formula, $\frac{n}{P} = 140$, gives an approximate relationship between *n* and *P* where,

n = number of steps per minute, and

P = pacelength in metres.

QUESTION 2.1

If the formula applies to Heiko's walking and Heiko takes 70 steps per minute, what is Heiko's pacelength? Show your work.

QUESTION 2.2

Bernard knows his pacelength is 0.80 metres. The formula applies to Bernard's walking.

Calculate Bernard's walking speed in metres per minute and in kilometres per hour. Show your working out.

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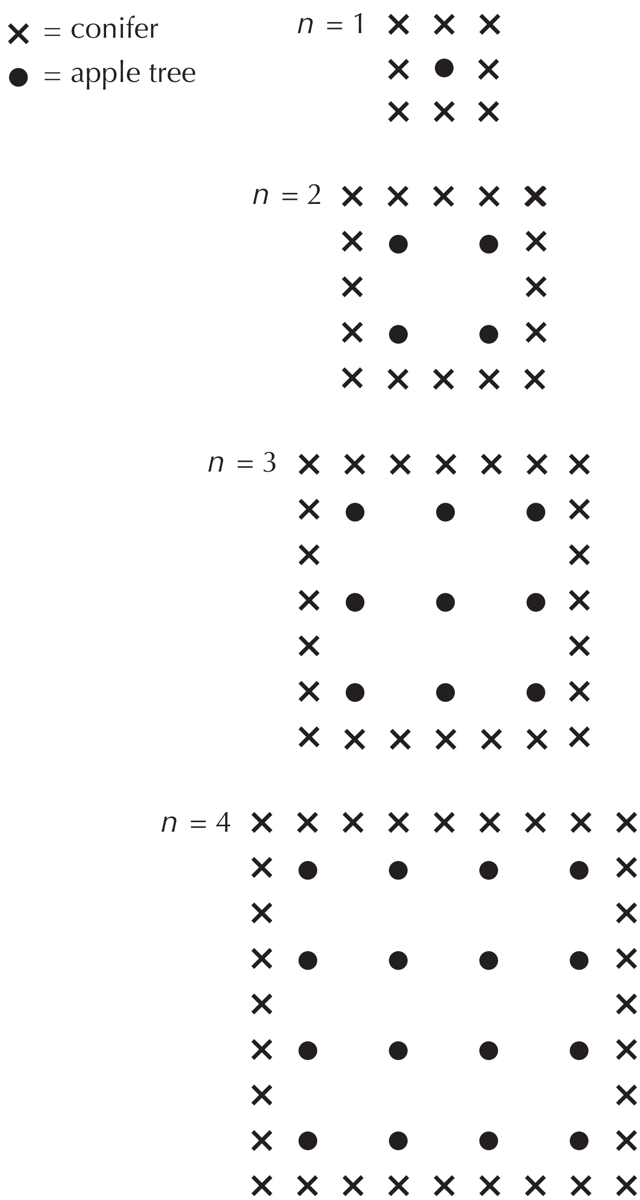
MATHEMATICS UNIT 3: APPLES

A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifer trees for any number (n) of rows of apple trees:



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QUESTION 3.1

Complete the table:

n	Number of apple trees	Number of conifer trees
1	1	8
2	4	
3		
4		
5		

QUESTION 3.2

There are two formulae you can use to calculate the number of apple trees and the number of conifer trees for the pattern described on the previous page:

Number of apple trees = n^2

Number of conifer trees = $8n$

where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifer trees. Find the value of n and show your method of calculating this.

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QUESTION 3.3

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifer trees? Explain how you found your answer.

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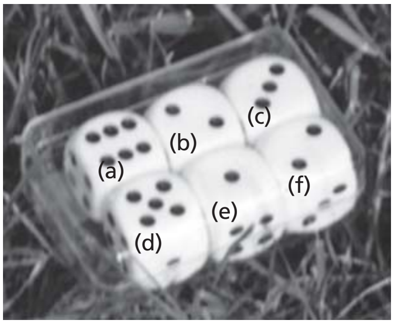
MATHEMATICS UNIT 4: CUBES

QUESTION 4.1

In this photograph you see six dice, labelled (a) to (f). For all dice there is a rule:

The total number of dots on two opposite faces of each die is always seven.

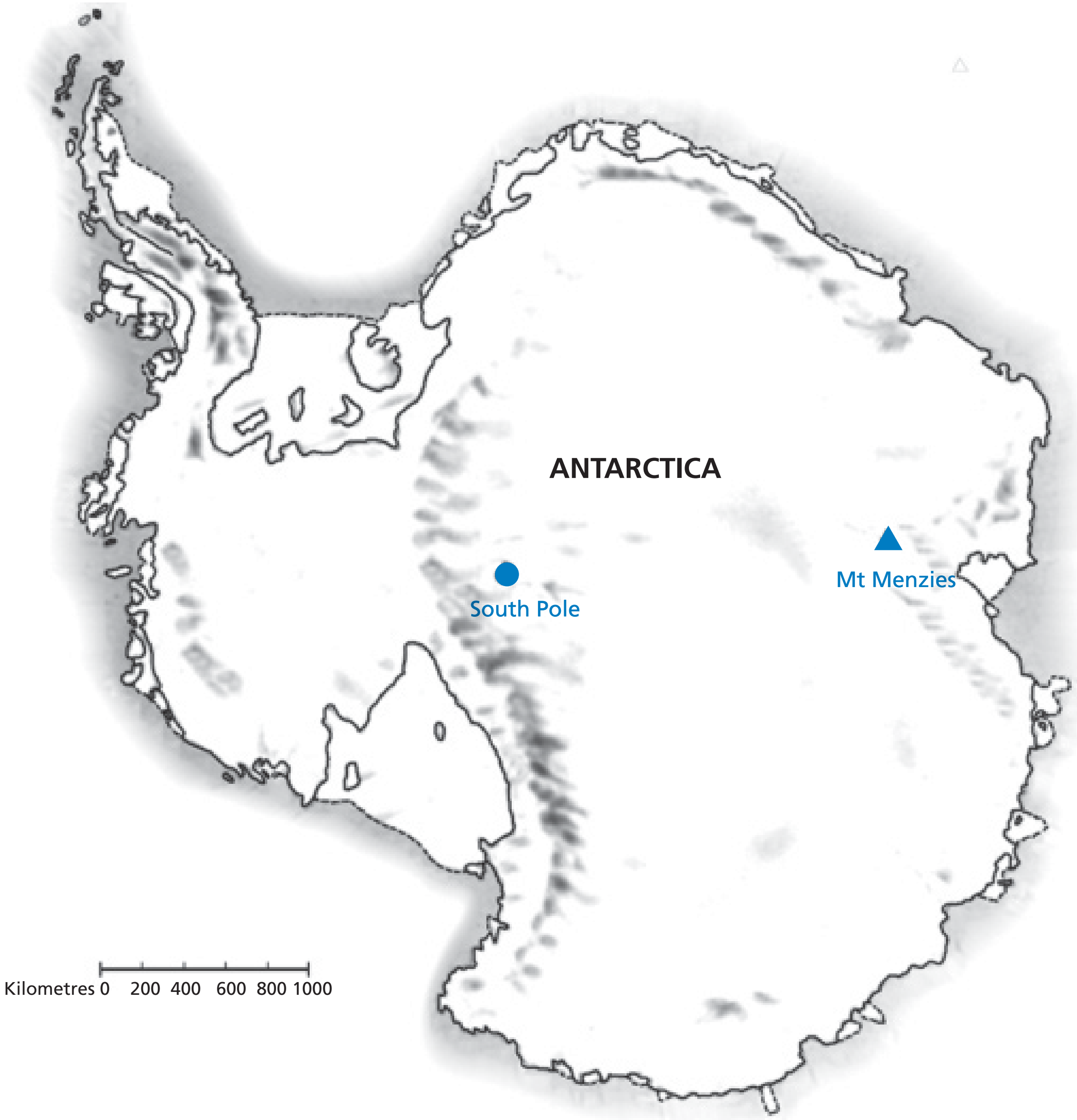
Write in each box the number of dots on the *bottom* face of the dice corresponding to the photograph.



(a)	(b)	(c)
(d)	(e)	(f)

MATHEMATICS UNIT 5: CONTINENT AREA

Below is a map of Antarctica.



QUESTION 5.1

Estimate the area of Antarctica using the map scale.

Show your working out and explain how you made your estimate. (You can draw over the map if it helps you with your estimation)

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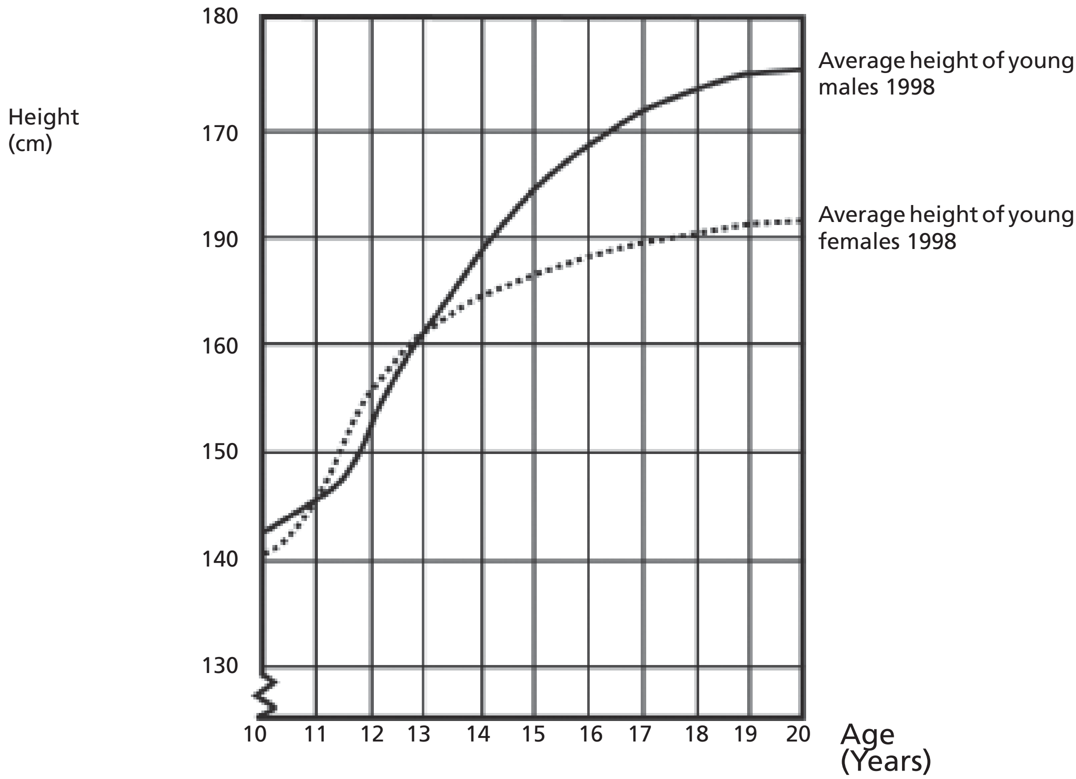
MATHEMATICS UNIT 6: GROWING UP

Youth grows taller

In 1998 the average height of both young males and young females in the Netherlands is represented in this graph.



QUESTION 6.1

Since 1980 the average height of 20-year-old females has increased by 2.3 cm, to 170.6 cm. What was the average height of a 20-year-old female in 1980?

Answer: ________________cm

QUESTION 6.2

Explain how the graph shows that on average the growth rate for girls slows down after 12 years of age.

....

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QUESTION 6.3

According to this graph, on average, during which period in their life are females taller than males of the same age?

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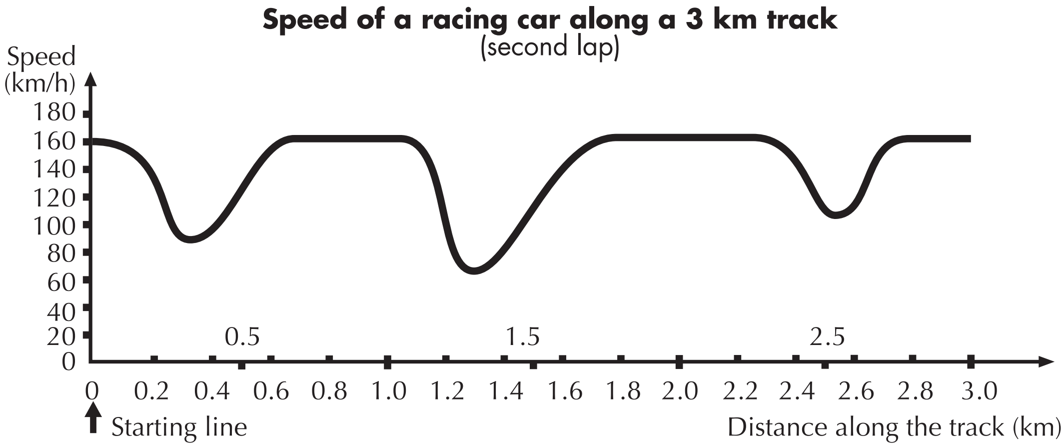
MATHEMATICS UNIT 7: SPEED OF RACING CAR

This graph shows how the speed of a racing car varies along a flat 3 kilometre track during its second lap.



Note: In memory of Claude Janvier, who died in June 1998. Modified task after his ideas in Janvier, C. (1978): *The interpretation of complex graphs – studies and teaching experiments*. Accompanying brochure to the Dissertation. University of Nottingham, Shell Centre for Mathematical Education, Item C-2. The pictures of the tracks are taken from Fischer, R. & Malle, G. (1985): *Mensch und Mathematik*. Bibliographisches Institut: Mannheim-Wien-Zurich, 234-238.

QUESTION 7.1

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A. 0.5 km

B. 1.5 km

C. 2.3 km

D. 2.6 km

QUESTION 7.2

Where was the lowest speed recorded during the second lap?

A. at the starting line.

B. at about 0.8 km.

C. at about 1.3 km.

D. halfway around the track.

QUESTION 7.3

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?

A. The speed of the car remains constant.

B. The speed of the car is increasing.

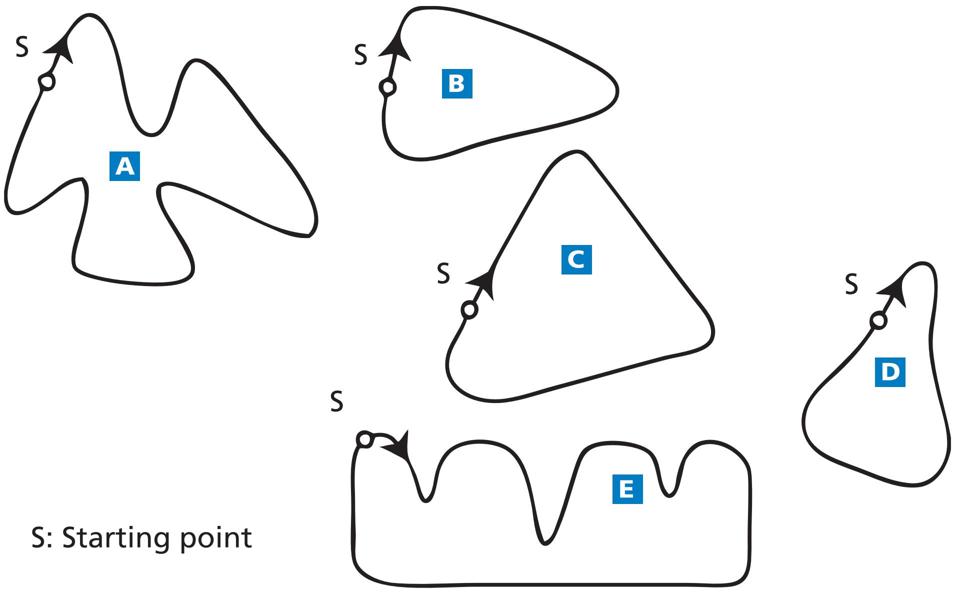
C. The speed of the car is decreasing.

D. The speed of the car cannot be determined from the graph.

QUESTION 7.4

Here are pictures of five tracks:

Along which one of these tracks was the car driven to produce the speed graph shown earlier?



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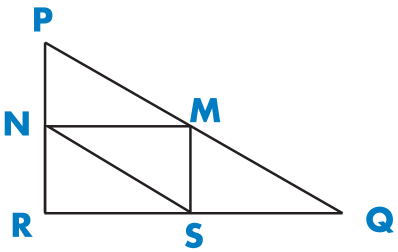
MATHEMATICS UNIT 8: TRIANGLES

QUESTION 8.1

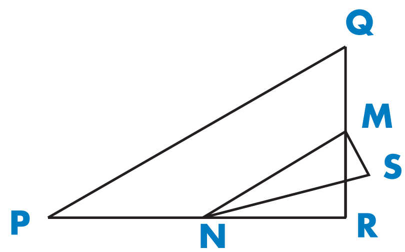
Circle the one figure below that fits the following description.

Triangle PQR is a right triangle with right angle at R. The line RQ is less than the line PR. M is the midpoint of the line PQ and N is the midpoint of the line QR. S is a point inside the triangle. The line MN is greater than the line MS.

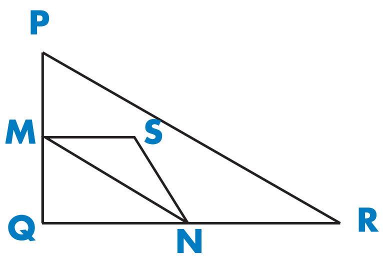
A



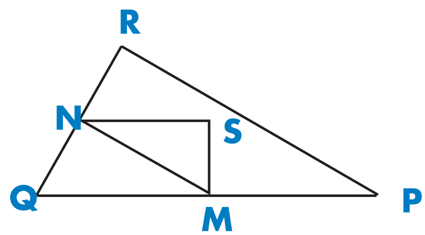
B



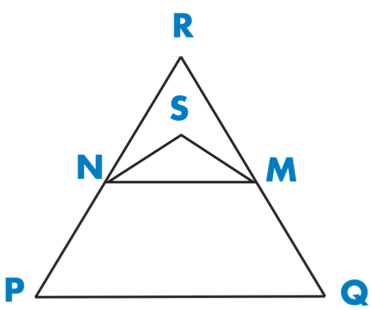
C



D



E

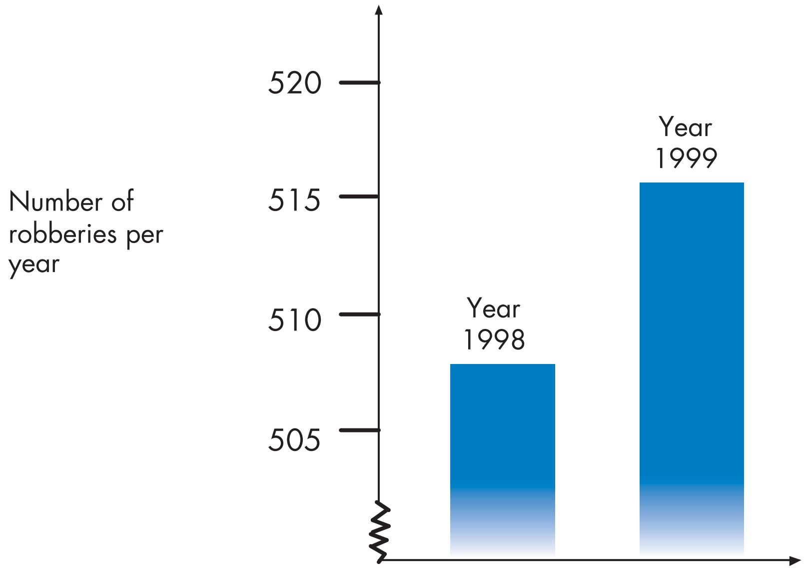


MATHEMATICS UNIT 9: ROBBERIES

QUESTION 9.1

A TV reporter showed this graph and said:

"The graph shows that there is a huge increase in the number of robberies from 1998 to 1999."



Do you consider the reporter's statement to be a reasonable interpretation of the graph? Give an explanation to support your answer.

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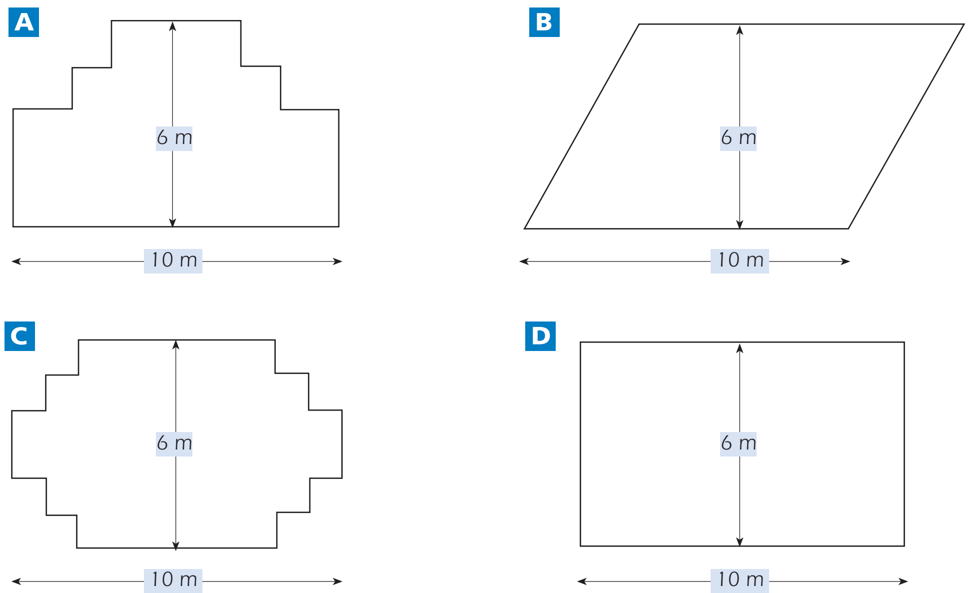
MATHEMATICS UNIT 10: CARPENTER

QUESTION 10.1

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

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MATHEMATICS UNIT 11: INTERNET RELAY CHAT

Mark (from Sydney, Australia) and Hans (from Berlin, Germany) often communicate with each other using "chat" on the Internet. They have to log on to the Internet at the same time to be able to chat.

To find a suitable time to chat, Mark looked up a chart of world times and found the following:

QUESTION 11.1

At 7:00 PM in Sydney, what time is it in Berlin?

Answer: ....

QUESTION 11.2

Mark and Hans are not able to chat between 9:00 AM and 4:30 PM their local time, as they have to go to school. Also, from 11:00 PM till 7:00 AM their local time they won't be able to chat because they will be sleeping.

When would be a good time for Mark and Hans to chat? Write the local times in the table.

Place	Time
Sydney	
Berlin	

MATHEMATICS UNIT 12: EXCHANGE RATE

Mei-Ling from Singapore was preparing to go to South Africa for 3 months as an exchange student. She needed to change some Singapore dollars (SGD) into South African rand (ZAR).

QUESTION 12.1

Mei-Ling found out that the exchange rate between Singapore dollars and South African rand was:

1 SGD = 4.2 ZAR

Mei-Ling changed 3000 Singapore dollars into South African rand at this exchange rate.

How much money in South African rand did Mei-Ling get?

Answer: ....

QUESTION 12.2

On returning to Singapore after 3 months, Mei-Ling had 3 900 ZAR left. She changed this back to Singapore dollars, noting that the exchange rate had changed to:

1 SGD = 4.0 ZAR

How much money in Singapore dollars did Mei-Ling get?

Answer: ....

QUESTION 12.3

During these 3 months the exchange rate had changed from 4.2 to 4.0 ZAR per SGD.

Was it in Mei-Ling's favour that the exchange rate now was 4.0 ZAR instead of 4.2 ZAR, when she changed her South African rand back to Singapore dollars? Give an explanation to support your answer.

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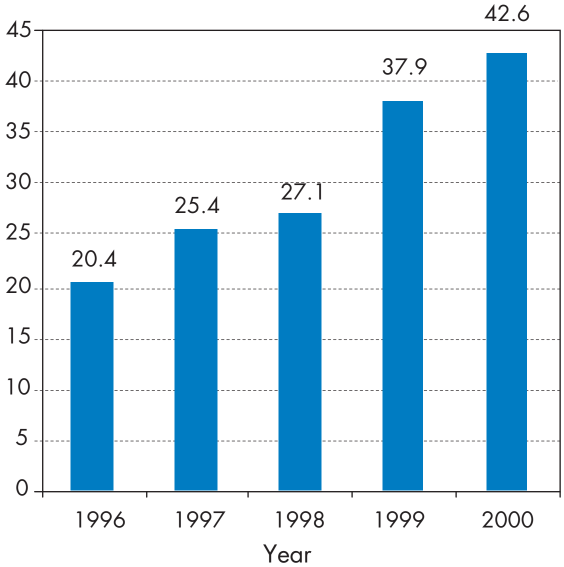
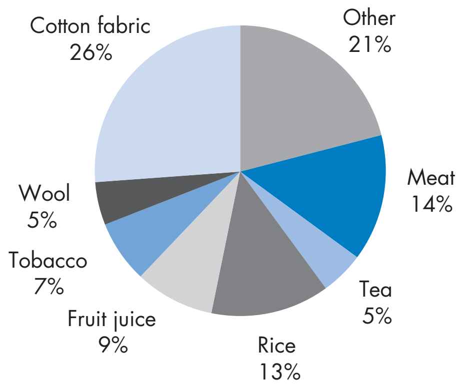
MATHEMATICS UNIT 13: EXPORTS

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.

Total annual exports from Zedland in millions of zeds, 1996 to 2000



Distribution of exports from Zedland in 2000



QUESTION 13.1

What was the total value (in millions of zeds) of exports from Zedland in 1998?

Answer: ....

QUESTION 13.2

What was the value of fruit juice exported from Zedland in 2000?

A. 1.8 million zeds.

B. 2.3 million zeds.

C. 2.4 million zeds.

D. 3.4 million zeds.

E. 3.8 million zeds.

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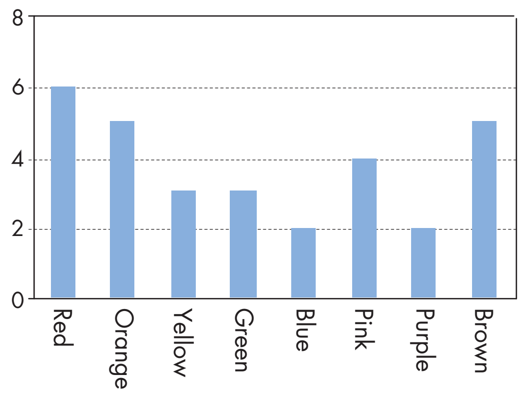
MATHEMATICS UNIT 14: COLOURED CANDIES



QUESTION 14.1

Robert's mother lets him pick one candy from a bag. He can't see the candies. The number of candies of each colour in the bag is shown in the following graph.

What is the probability that Robert will pick a red candy?

A. 10%

B. 20%

C. 25%

D. 50%

MATHEMATICS UNIT 15: SCIENCE TESTS

QUESTION 15.1

In Mei Lin's school, her science teacher gives tests that are marked out of 100. Mei Lin has an average of 60 marks on her first four Science tests. On the fifth test she got 80 marks.

What is the average of Mei Lin's marks in Science after all five tests?

Average: ....

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MATHEMATICS UNIT 16: BOOKSHELVES

QUESTION 16.1

To complete one set of bookshelves a carpenter needs the following components:

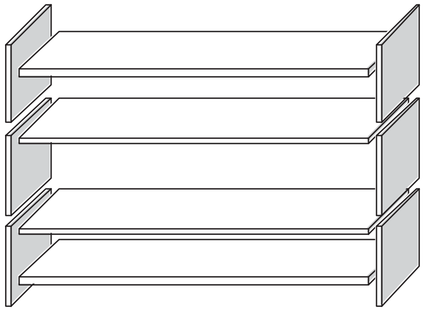
4 long wooden panels,

6 short wooden panels,

12 small clips,

2 large clips and

14 screws.



The carpenter has in stock 26 long wooden panels, 33 short wooden panels, 200 small clips, 20 large clips and 510 screws.

How many sets of bookshelves can the carpenter make?

Answer: ....

MATHEMATICS UNIT 17: LITTER

QUESTION 17.1

For a homework assignment on the environment, students collected information on the decomposition time of several types of litter that people throw away:

Type of Litter	***Decomposition time***
Banana peel	1–3 years
Orange peel	1–3 years
Cardboard boxes	0.5 year
Chewing gum	20–25 years
Newspapers	A few days
Polystyrene cups	Over 100 years

A student thinks of displaying the results in a bar graph.

Give *one* reason why a bar graph is unsuitable for displaying these data.

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MATHEMATICS UNIT 18: EARTHQUAKE

QUESTION 18.1

A documentary was broadcast about earthquakes and how often earthquakes occur. It included a discussion about the predictability of earthquakes.

A geologist stated: “In the next twenty years, the chance that an earthquake will occur in Zed City is two out of three”.

Which of the following best reflects the meaning *of the geologist’s statement*?

A. $\frac{2}{3} \times 20 = 13.3$, so between 13 and 14 years from now there will be an earthquake in Zed City.

B. $\frac{2}{3}$ is more than $\frac{1}{2}$, so you can be sure there will be an earthquake in Zed City at some time during the next 20 years.

C. The likelihood that there will be an earthquake in Zed City at some time during the next 20 years is higher than the likelihood of no earthquake.

D. You cannot tell what will happen, because nobody can be sure when an earthquake will occur.

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MATHEMATICS UNIT 19: CHOICES

QUESTION 19.1

In a pizza restaurant, you can get a basic pizza with two toppings: cheese and tomato. You can also make up your own pizza with **extra** toppings. You can choose from four different extra toppings: olives, ham, mushrooms and salami.

Ross wants to order a pizza with two different **extra** toppings.

How many different combinations can Ross choose from?

Answer: .... combinations.

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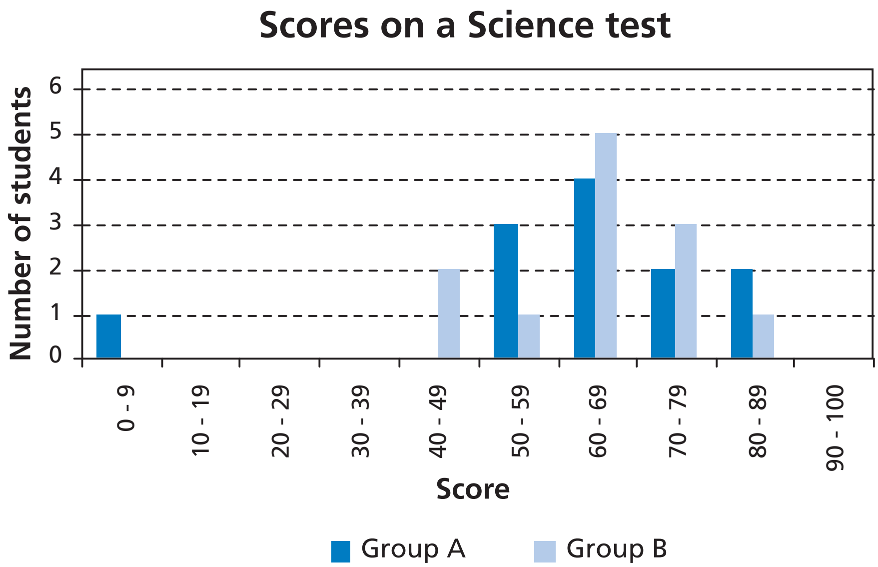
MATHEMATICS UNIT 20: TEST SCORES

QUESTION 20.1

The diagram below shows the results on a Science test for two groups, labelled as Group A and Group B.

The mean score for Group A is 62.0 and the mean for Group B is 64.5. Students pass this test when their score is 50 or above.



Looking at the diagram, the teacher claims that Group B did better than Group A in this test.

The students in Group A don't agree with their teacher. They try to convince the teacher that Group B may not necessarily have done better.

Give one mathematical argument, using the graph, that the students in Group A could use.

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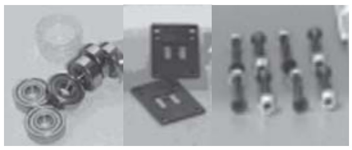
MATHEMATICS UNIT 21 : SKATEBOARD

Eric is a great skateboard fan. He visits a shop named SKATERS to check some prices.

At this shop you can buy a complete board. Or you can buy a deck, a set of 4 wheels, a set of 2 trucks and a set of hardware, and assemble your own board.

The prices for the shop's products are:

Product	Price in zeds	
Complete skateboard	82 or 84	
Deck	40, 60 or 65	
One set of 4 Wheels	14 or 36	
One set of 2 Trucks	16	
One set of hardware (bearings, rubber pads, bolts and nuts)	10 or 20	

QUESTION 21.1

Eric wants to assemble his own skateboard. What is the minimum price and the maximum price in this shop for self-assembled skateboards?

(a) Minimum price:............ zeds.

(b) Maximum price:............ zeds.

QUESTION 21.2

The shop offers three different decks, two different sets of wheels and two different sets of hardware. There is only one choice for a set of trucks.

How many different skateboards can Eric construct?

A. 6

B. 8

C. 10

D. 12

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QUESTION 21.3

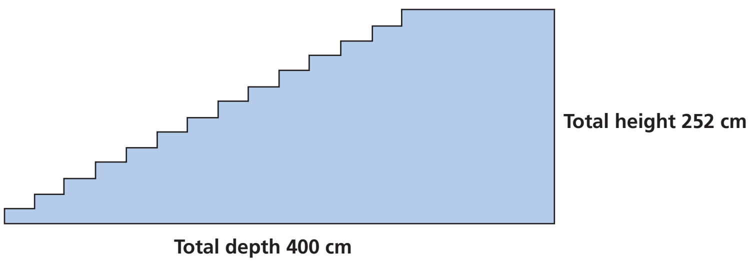
Eric has 120 zeds to spend and wants to buy the most expensive skateboard he can afford.

How much money can Eric afford to spend on each of the 4 parts? Put your answer in the table below.

Part	Amount (zeds)
Deck	
Wheels	
Trucks	
Hardware	

MATHEMATICS UNIT 22: STAIRCASE

QUESTION 22.1



The diagram above illustrates a staircase with 14 steps and a total height of 252 cm:

What is the height of each of the 14 steps?

Height: ....cm.

MATHEMATICS UNIT 23 : NUMBER CUBES

QUESTION 23.1

On the right, there is a picture of two dice.

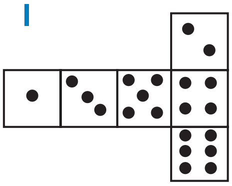
Dice are special number cubes for which the following rule applies:

The total number of dots on two opposite faces is always seven.

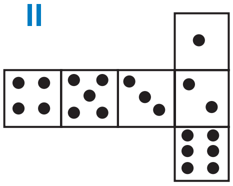
You can make a simple number cube by cutting, folding and gluing cardboard. This can be done in many ways. In the figure below you can see four cuttings that can be used to make cubes, with dots on the sides.

Which of the following shapes can be folded together to form a cube that obeys the rule that the sum of opposite faces is 7? For each shape, circle either "Yes" or "No" in the table below.

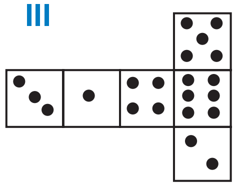
I



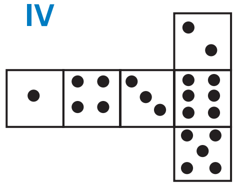
II



III



IV



Shape	Obeys the rule that the sum of opposite faces is 7?
I	Yes / No
II	Yes / No
III	Yes / No
IV	Yes / No

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MATHEMATICS UNIT 24: SUPPORT FOR THE PRESIDENT

QUESTION 24.1

In Zedland, opinion polls were conducted to find out the level of support for the President in the forthcoming election. Four newspaper publishers did separate nationwide polls. The results for the four newspaper polls are shown below:

Newspaper 1: 36.5% (poll conducted on January 6, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 2: 41.0% (poll conducted on January 20, with a sample of 500 randomly selected citizens with voting rights)

Newspaper 3: 39.0% (poll conducted on January 20, with a sample of 1000 randomly selected citizens with voting rights)

Newspaper 4: 44.5% (poll conducted on January 20, with 1000 readers phoning in to vote).

Which newspaper's result is likely to be the best for predicting the level of support for the President if the election is held on January 25? Give two reasons to support your answer.

....

....

....

....

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MATHEMATICS UNIT 25: THE BEST CAR

A car magazine uses a rating system to evaluate new cars, and gives the award of "The Car of the Year" to the car with the highest total score. Five new cars are being evaluated, and their ratings are shown in the table.

Car	Safety Features (S)	Fuel Efficiency (F)	External Appearance (E)	Internal Fittings (T)
Ca	3	1	2	3
M2	2	2	2	2
Sp	3	1	3	2
N1	1	3	3	3
KK	3	2	3	2

The ratings are interpreted as follows:

3 points = Excellent

2 points = Good

1 point = Fair

QUESTION 25.1

To calculate the total score for a car, the car magazine uses the following rule, which is a weighted sum of the individual score points:

Total Score = (3 x S) + F + E + T

Calculate the total score for Car "Ca". Write your answer in the space below.

Total score for "Ca": ....

QUESTION 25.2

The manufacturer of car "Ca" thought the rule for the total score was unfair.

Write down a rule for calculating the total score so that Car "Ca" will be the winner.

Your rule should include all four of the variables, and you should write down your rule by filling in positive numbers in the four spaces in the equation below.

Total score = ...x S + ... x F + ... x E + ... x T.

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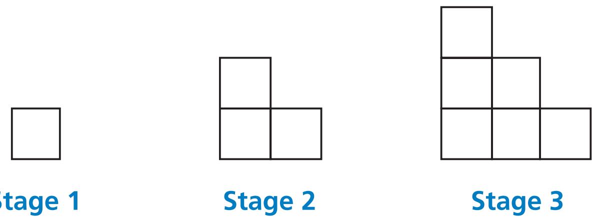
MATHEMATICS UNIT 26: STEP PATTERN

QUESTION 26.1

Robert builds a step pattern using squares. Here are the stages he follows.



As you can see, he uses one square for Stage 1, three squares for Stage 2 and six for Stage 3.

How many squares should he use for the fourth stage?

Answer: ....squares.

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MATHEMATICS UNIT 27 : LICHEN

A result of global warming is that the ice of some glaciers is melting. Twelve years after the ice disappears, tiny plants, called lichen, start to grow on the rocks.

Each lichen grows approximately in the shape of a circle.

The relationship between the diameter of this circle and the age of the lichen can be approximated with the formula:

$$d = 7.0 \times \sqrt{(t - 12)} \quad \text{for } t \geq 12$$

where *d* represents the diameter of the lichen in millimetres, and *t* represents the number of years after the ice has disappeared.

QUESTION 27.1

Using the formula, calculate the diameter of the lichen, 16 years after the ice disappeared.

Show your calculation.

....

....

QUESTION 27.2

Ann measured the diameter of some lichen and found it was 35 millimetres.

How many years ago did the ice disappear at this spot?

Show your calculation.

....

....

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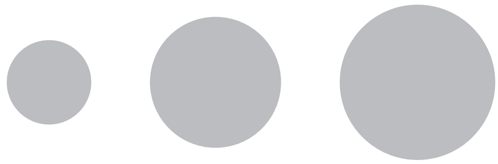
MATHEMATICS UNIT 28: COINS

You are asked to design a new set of coins. All coins will be circular and coloured silver, but of different diameters.



Researchers have found out that an ideal coin system meets the following requirements:

- diameters of coins should not be smaller than 15 mm and not be larger than 45 mm.
- given a coin, the diameter of the next coin must be at least 30% larger.
- the minting machinery can only produce coins with diameters of a whole number of millimetres (*e.g.* 17 mm is allowed, 17.3 mm is not).

QUESTION 28.1

You are asked to design a set of coins that satisfy the above requirements.
You should start with a 15 mm coin and your set should contain as many coins as possible. What would be the diameters of the coins in your set?

....

....

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MATHEMATICS UNIT 29 : PIZZAS

A pizzeria serves two round pizzas of the same thickness in different sizes. The smaller one has a diameter of 30 cm and costs 30 zeds. The larger one has a diameter of 40 cm and costs 40 zeds.

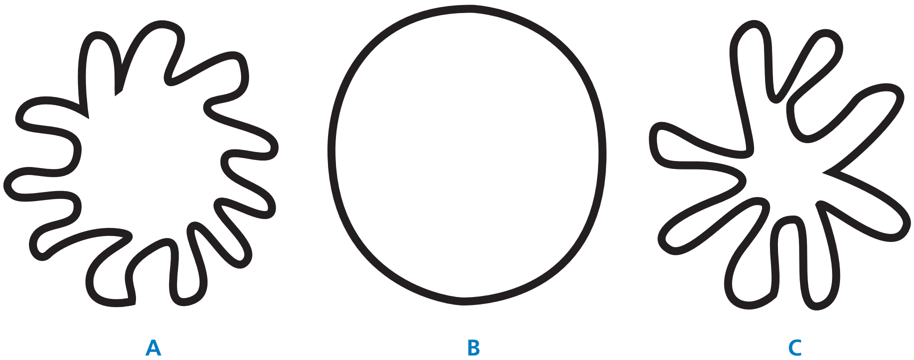
QUESTION 29.1

Which pizza is better value for money? Show your reasoning.

....

....

MATHEMATICS UNIT 30: SHAPES



QUESTION 30.1

Which of the figures has the largest area? Explain your reasoning.

....

....

QUESTION 30.2

Describe a method for estimating the area of figure C.

....

....

QUESTION 30.3

Describe a method for estimating the perimeter of figure C.

....

....

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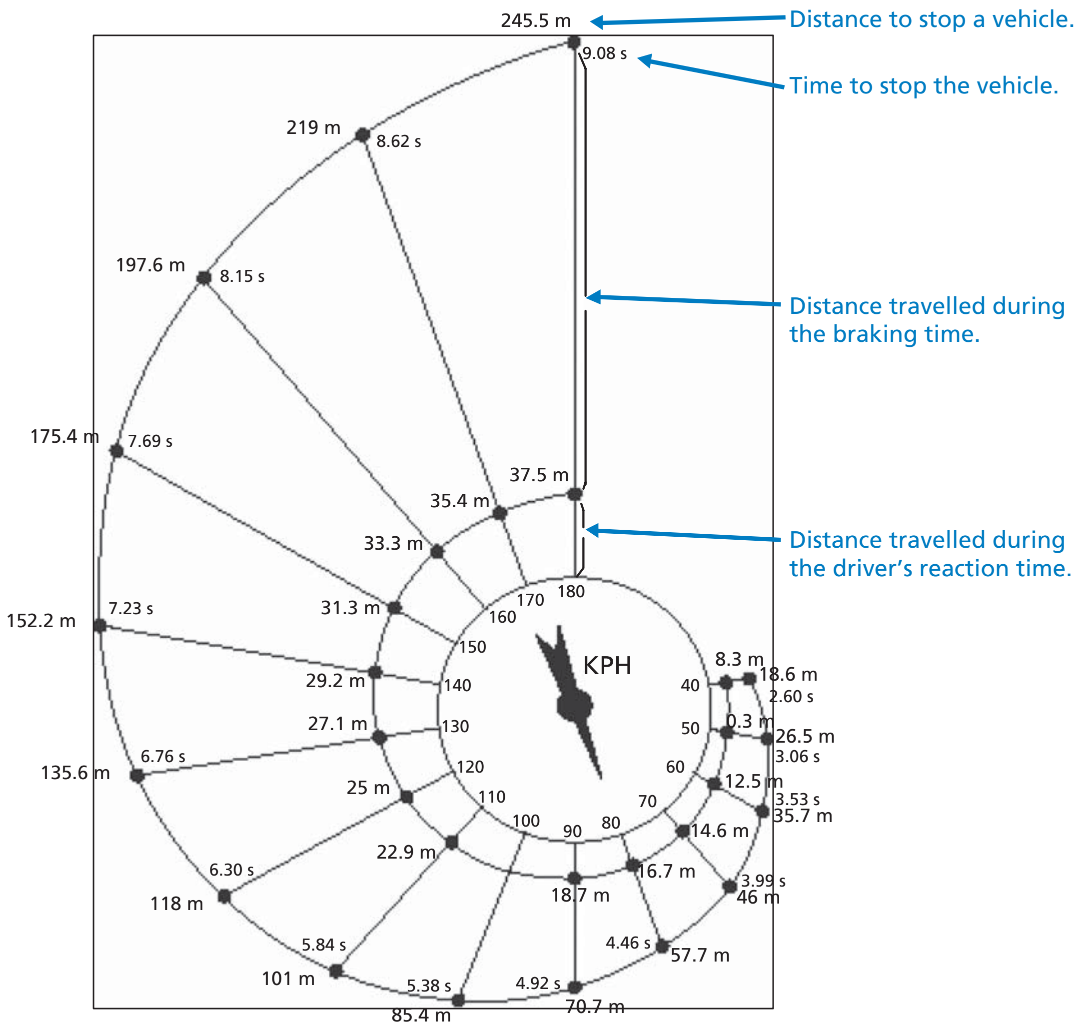
MATHEMATICS UNIT 31 : BRAKING

The approximate distance to stop a moving vehicle is the sum of:

- the distance covered during the time the driver takes to begin to apply the brakes (reaction-time distance)
- the distance travelled while the brakes are applied (braking distance)

The 'snail' diagram below gives the theoretical stopping distance for a vehicle in good braking conditions (a particularly alert driver, brakes and tyres in perfect condition, a dry road with a good surface) and how much the stopping distance depends on speed.



Source: La Prévention Routière, Ministère de l'Education nationale, de la Recherche et de la Technologie, France.

QUESTION 31.1

If a vehicle is travelling at 110 kph, what distance does the vehicle travel during the driver's reaction time?

....

....

QUESTION 31.2

If a vehicle is travelling at 110 kph, what is the total distance travelled before the vehicle stops?

....

....

QUESTION 31.3

If a vehicle is travelling at 110 kph, how long does it take to stop the vehicle completely?

....

....

QUESTION 31.4

If a vehicle is travelling at 110 kph, what is the distance travelled while the brakes are being applied?

....

....

QUESTION 31.5

A second driver, travelling in good conditions, stops her vehicle in a total distance of 70.7 metres.
At what speed was the vehicle travelling before the brakes were applied?

....

....

MATHEMATICS UNIT 32 : PATIO

QUESTION 32.1

Nick wants to pave the rectangular patio of his new house. The patio has length 5.25 metres and width 3.00 metres. He needs 81 bricks per square metre.

Calculate how many bricks Nick needs for the whole patio.

....

....

....

MATHEMATICS UNIT 33: DRUG CONCENTRATIONS

QUESTION 33.1

A woman in hospital receives an injection of penicillin. Her body gradually breaks the penicillin down so that one hour after the injection only 60% of the penicillin will remain active.

This pattern continues: at the end of each hour only 60% of the penicillin that was present at the end of the previous hour remains active.

Suppose the woman is given a dose of 300 milligrams of penicillin at 8 o'clock in the morning.

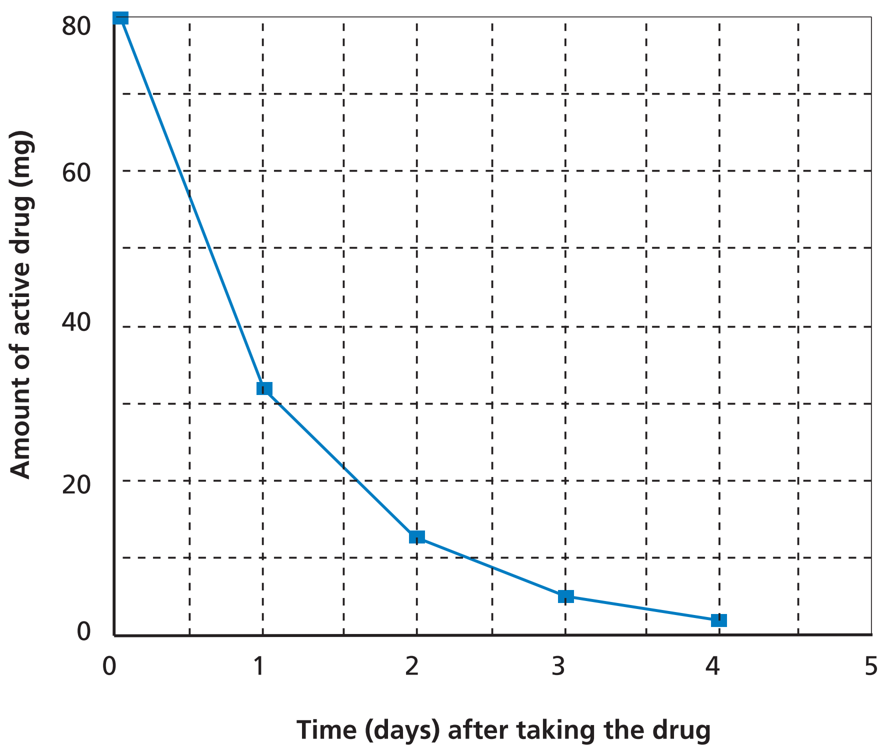
3

Complete this table showing the amount of penicillin that will remain active in the woman's blood at intervals of one hour from 0800 until 1100 hours.

Time	0800	0900	1000	1100
Penicillin (mg)	300			

QUESTION 33.2

Peter has to take 80 mg of a drug to control his blood pressure. The following graph shows the initial amount of the drug, and the amount that remains active in Peter's blood after one, two, three and four days.



How much of the drug remains active at the end of the first day?

A. 6 mg.

B. 12 mg.

C. 26 mg.

D. 32 mg.

QUESTION 33.3

From the graph for the previous question it can be seen that each day, about the same proportion of the previous day's drug remains active in Peter's blood.

At the end of each day which of the following is the approximate percentage of the previous day's drug that remains active?

A. 20%.

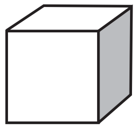
B. 30%.

C. 40%.

D. 80%.

MATHEMATICS UNIT 34: BUILDING BLOCKS

Susan likes to build blocks from small cubes like the one shown in the following diagram:



Small cube

3

Susan has lots of small cubes like this one. She uses glue to join cubes together to make other blocks.

First, Susan glues eight of the cubes together to make the block shown in Diagram A:

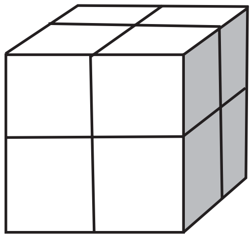


Diagram A

Then Susan makes the solid blocks shown in Diagram B and Diagram C below:

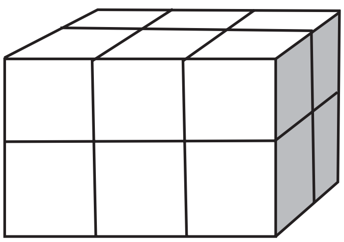


Diagram B

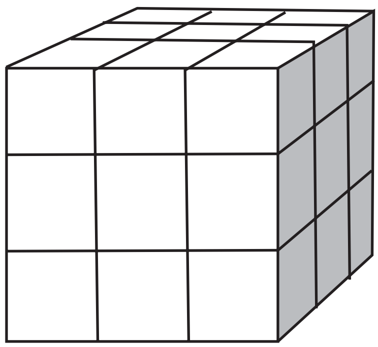


Diagram C

QUESTION 34.1

How many small cubes will Susan need to make the block shown in Diagram B?

Answer: .... cubes.

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QUESTION 34.2

How many small cubes will Susan need to make the solid block shown in Diagram C?

Answer: .... cubes.

QUESTION 34.3

Susan realises that she used more small cubes than she really needed to make a block like the one shown in Diagram C. She realises that she could have glued small cubes together to look like Diagram C, but the block could have been hollow on the inside.

What is the minimum number of cubes she needs to make a block that looks like the one shown in Diagram C, but is hollow?

Answer: .... cubes.

QUESTION 34.4

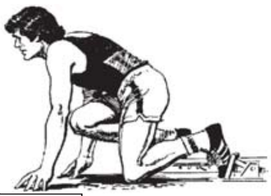
Now Susan wants to make a block that looks like a solid block that is 6 small cubes long, 5 small cubes wide and 4 small cubes high. She wants to use the smallest number of cubes possible, by leaving the largest possible hollow space inside the block.

What is the minimum number of cubes Susan will need to make this block?

Answer: .... cubes.

MATHEMATICS UNIT 35: REACTION TIME

In a Sprinting event, the 'reaction time' is the time interval between the starter's gun firing and the athlete leaving the starting block. The 'final time' includes both this reaction time, and the running time.



The following table gives the reaction time and the final time of 8 runners in a 100 metre sprint race.

Lane	Reaction time (sec)	Final time (sec)
1	0.147	10.09
2	0.136	9.99
3	0.197	9.87
4	0.180	Did not finish the race
5	0.210	10.17
6	0.216	10.04
7	0.174	10.08
8	0.193	10.13

QUESTION 35.1

Identify the Gold, Silver and Bronze medallists from this race. Fill in the table below with the medallists' lane number, reaction time and final time.

Medal	Lane	Reaction time (secs)	Final time (secs)
GOLD			
SILVER			
BRONZE			

QUESTION 35.2

To date, no humans have been able to react to a starter's gun in less than 0.110 second.

If the recorded reaction time for a runner is less than 0.110 second, then a false start is considered to have occurred because the runner must have left before hearing the gun.

If the Bronze medallist had a faster reaction time, would he have had a chance to win the Silver medal? Give an explanation to support your answer.

....

....

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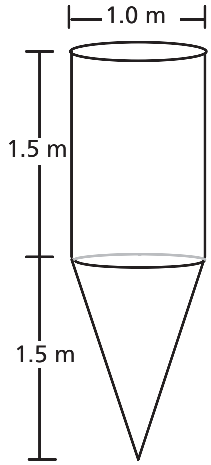
MATHEMATICS UNIT 36: WATER TANK

QUESTION 36.1

A water tank has shape and dimensions as shown in the diagram.

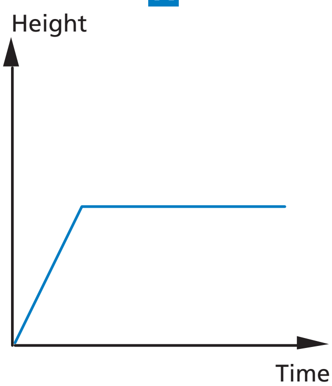
At the beginning the tank is empty. Then it is filled with water at the rate of one litre per second.



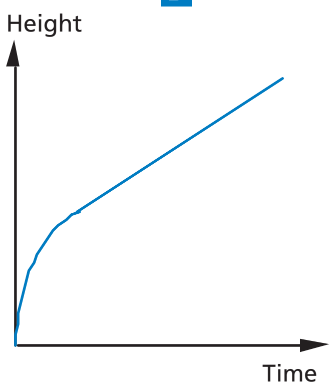
Water tank

Which of the following graphs shows how the height of the water surface changes over time?

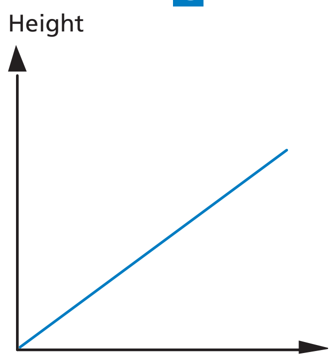
A



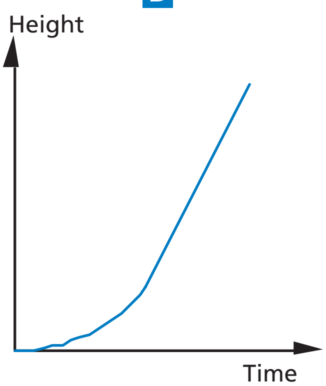
B



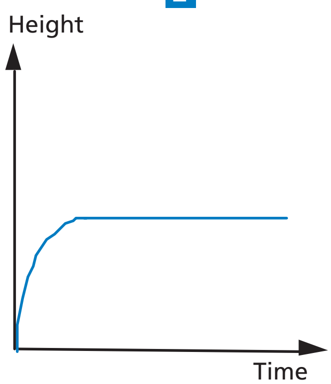
C



D



E



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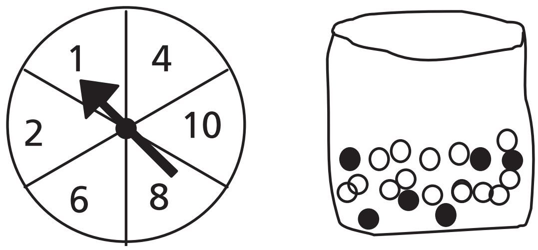
MATHEMATICS UNIT 37: SPRING FAIR

QUESTION 37.1

A game in a booth at a spring fair involves using a spinner first. Then, **if** the spinner stops on an even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in the diagram below.



Prizes are given when a black marble is picked. Sue plays the game once.

How likely is it that Sue will win a prize?

A. Impossible.

B. Not very likely.

C. About 50% likely.

D. Very likely.

E. Certain.

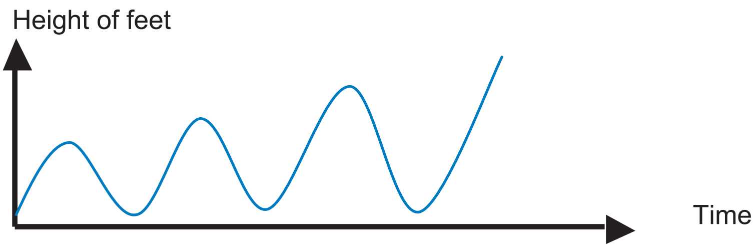
MATHEMATICS UNIT 38: SWING

QUESTION 38.1

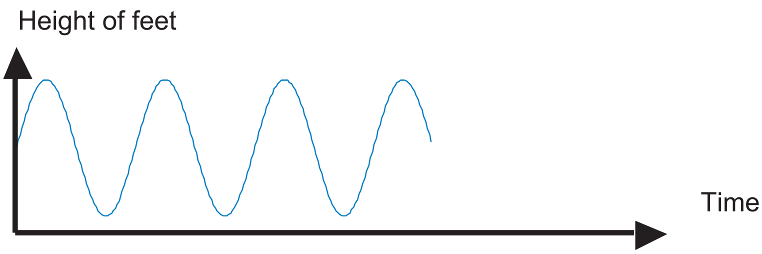
Mohammed is sitting on a swing. He starts to swing. He is trying to go as high as possible.

Which diagram best represents the height of his feet above the ground as he swings?

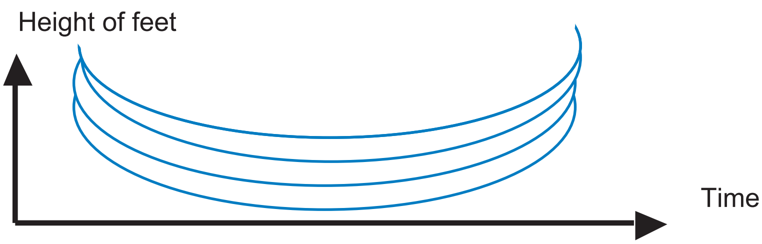
A



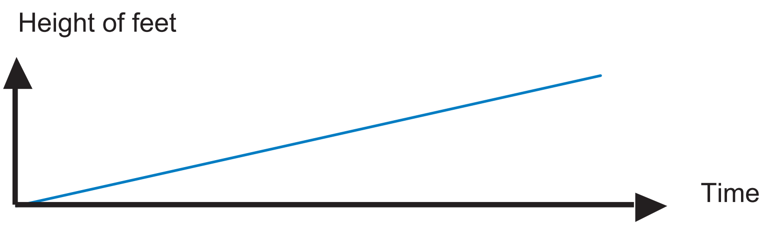
B



C



D



MATHEMATICS UNIT 39: STUDENT HEIGHTS

3

QUESTION 39.1

In a mathematics class one day, the heights of all students were measured. The average height of boys was 160 cm, and the average height of girls was 150 cm. Alena was the tallest – her height was 180 cm. Zdenek was the shortest – his height was 130 cm.

Two students were absent from class that day, but they were in class the next day. Their heights were measured, and the averages were recalculated. Amazingly, the average height of the girls and the average height of the boys did not change.

Which of the following conclusions can be drawn from this information?

Circle 'Yes' or 'No' for each conclusion.

Conclusion	Can this conclusion be drawn?
Both students are girls.	Yes / No
One of the students is a boy and the other is a girl.	Yes / No
Both students have the same height.	Yes / No
The average height of all students did not change.	Yes / No
Zdenek is still the shortest.	Yes / No

MATHEMATICS UNIT 40: PAYMENTS BY AREA

People living in an apartment building decide to buy the building. They will put their money together in such a way that each will pay an amount that is proportional to the size of their apartment.

For example, a man living in an apartment that occupies one fifth of the floor area of all apartments will pay one fifth of the total price of the building.

QUESTION 40.1

Circle Correct or Incorrect for each of the following statements.

Statement	Correct / Incorrect
A person living in the largest apartment will pay more money for each square metre of his apartment than the person living in the smallest apartment.	Correct / Incorrect
If we know the areas of two apartments and the price of one of them we can calculate the price of the second.	Correct / Incorrect
If we know the price of the building and how much each owner will pay, then the total area of all apartments can be calculated.	Correct / Incorrect
If the total price of the building were reduced by 10%, each of the owners would pay 10% less.	Correct / Incorrect

QUESTION 40.2

There are three apartments in the building. The largest, apartment 1, has a total area of $95m^2$. Apartments 2 and 3 have areas of $85m^2$ and $70m^2$ respectively. The selling price for the building is 300 000 zeds.

How much should the owner of apartment 2 pay? Show your work.

....

....

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MATHEMATICS UNIT 41 : SHOES FOR KIDS

The following table shows the recommended Zedland shoe sizes corresponding to various foot lengths.

Conversion table for kids shoe sizes in Zedland

From (in mm)	To (in mm)	Shoe size
107	115	18
116	122	19
123	128	20
129	134	21
135	139	22
140	146	23
147	152	24
153	159	25
160	166	26
167	172	27
173	179	28
180	186	29
187	192	30
193	199	31
200	206	32
207	212	33
213	219	34
220	226	35

QUESTION 41.1

Marina's feet are 163 mm long. Use the table to determine which Zedland shoe size Marina should try on.

Answer: ....

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MATHEMATICS UNIT 42 : TABLE TENNIS TOURNAMENT

QUESTION 42.1

Teun, Riek, Bep and Dirk have formed a practice group in a table tennis club. Each player wishes to play against each other player once. They have reserved two practice tables for these matches.

Complete the following match schedule; by writing the names of the players playing in each match.

	Practice Table 1	Practice Table 2
Round 1	Teun – Riek	Bep – Dirk
Round 2	.... –	.... – ....
Round 3	.... –	.... – ....

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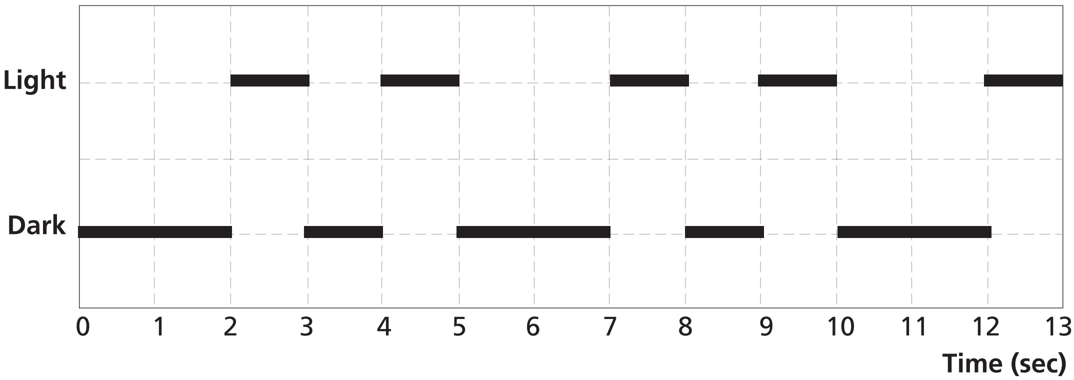
MATHEMATICS UNIT 43: LIGHTHOUSE

Lighthouses are towers with a light beacon on top. Lighthouses assist sea ships in finding their way at night when they are sailing close to the shore.

A lighthouse beacon sends out light flashes with a regular fixed pattern. Every lighthouse has its own pattern.

In the diagram below you see the pattern of a certain lighthouse. The light flashes alternate with dark periods.



It is a regular pattern. After some time the pattern repeats itself. The time taken by one complete cycle of a pattern, before it starts to repeat, is called the *period.* When you find the period of a pattern, it is easy to extend the diagram for the next seconds or minutes or even hours.

QUESTION 43.1

Which of the following could be the period of the pattern of this lighthouse?

A. 2 seconds.

B. 3 seconds.

C. 5 seconds.

D. 12 seconds.

QUESTION 43.2

For how many seconds does the lighthouse send out light flashes in 1 minute?

A. 4

B. 12

C. 20

D. 24

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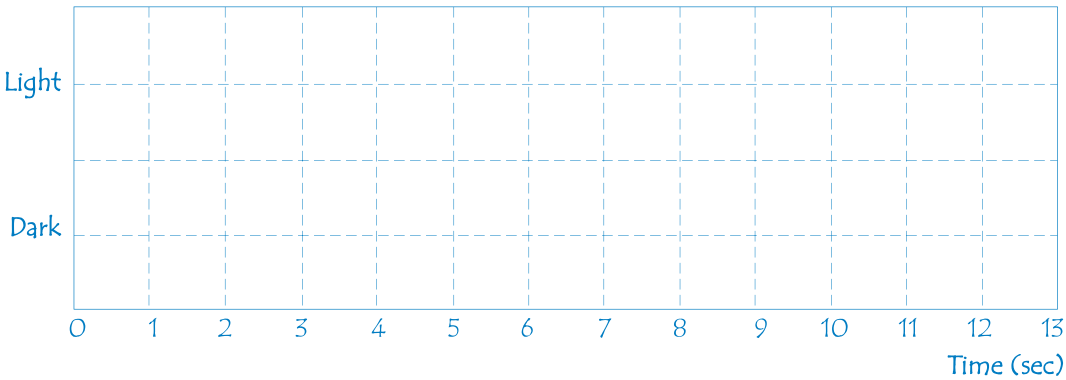
QUESTION 43.3

In the diagram below, make a graph of a possible pattern of light flashes of a lighthouse that sends out light flashes for 30 seconds per minute. The period of this pattern must be equal to 6 seconds.

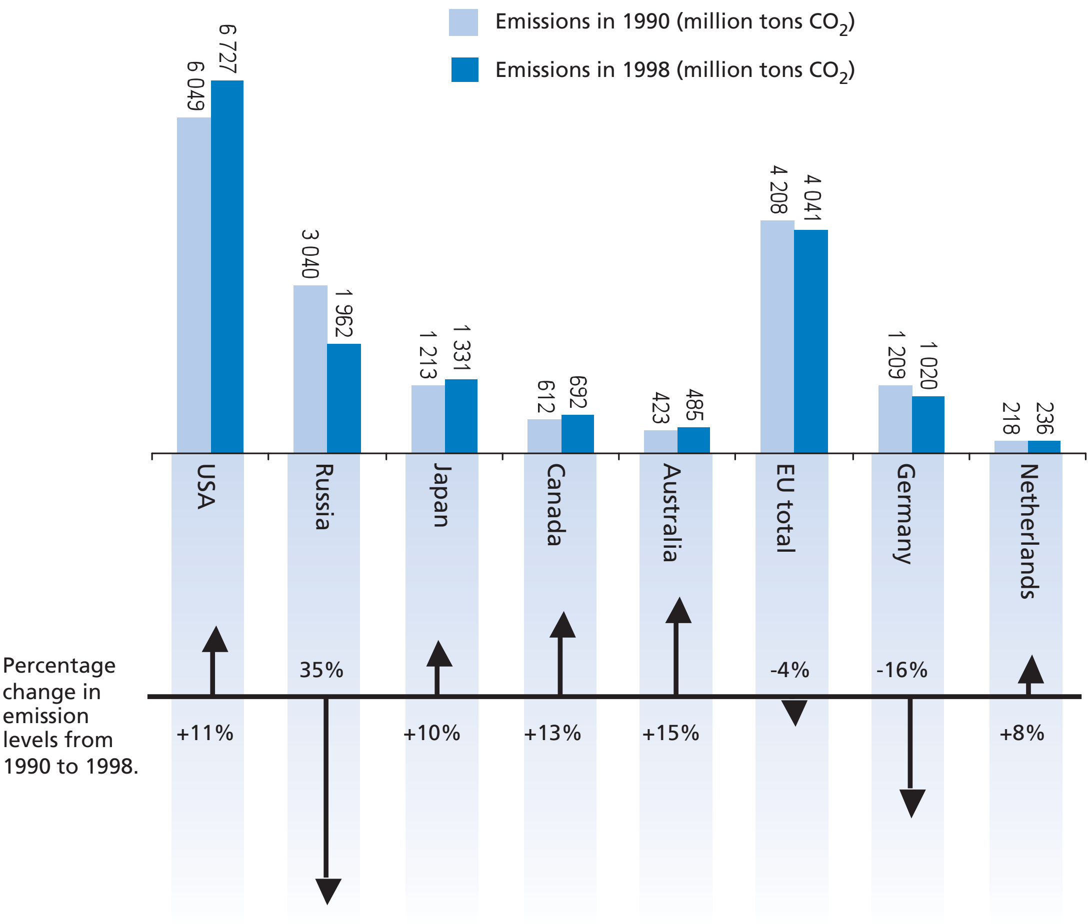


3

MATHEMATICS UNIT 44: DECREASING CO_2 LEVELS

Many scientists fear that the increasing level of CO_2 gas in our atmosphere is causing climate change.

The diagram below shows the CO_2 emission levels in 1990 (the light bars) for several countries (or regions), the emission levels in 1998 (the dark bars), and the percentage change in emission levels between 1990 and 1998 (the arrows with percentages).



QUESTION 44.1

In the diagram you can read that in the USA, the increase in CO_2 emission level from 1990 to 1998 was 11%.

Show the calculation to demonstrate how the 11% is obtained.

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QUESTION 44.2

Mandy analysed the diagram and claimed she discovered a mistake in the percentage change in emission levels: "The percentage decrease in Germany (16%) is bigger than the percentage decrease in the whole European Union (EU total, 4%). This is not possible, since Germany is part of the EU."

Do you agree with Mandy when she says this is not possible? Give an explanation to support your answer.

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QUESTION 44.3

Mandy and Niels discussed which country (or region) had the largest *increase* of CO_2 emissions.

Each came up with a different conclusion based on the diagram.

Give two possible 'correct' answers to this question, and explain how you can obtain each of these answers.

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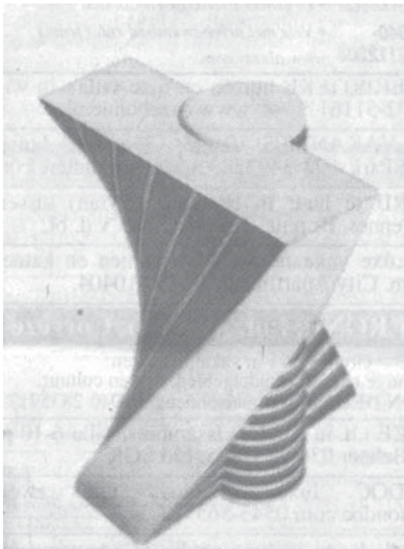
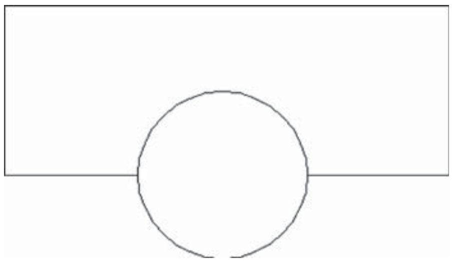
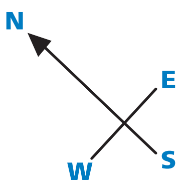
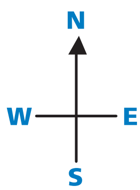
MATHEMATICS UNIT 45: TWISTED BUILDING

In modern architecture, buildings often have unusual shapes. The picture below shows a computer model of a 'twisted building' and a plan of the ground floor. The compass points show the orientation of the building.









The ground floor of the building contains the main entrance and has room for shops. Above the ground floor there are 20 storeys containing apartments.

The plan of each storey is similar to the plan of the ground floor, but each has a slightly different orientation from the storey below. The cylinder contains the elevator shaft and a landing on each floor.

QUESTION 45.1

Estimate the total height of the building, in metres. Explain how you found your answer.

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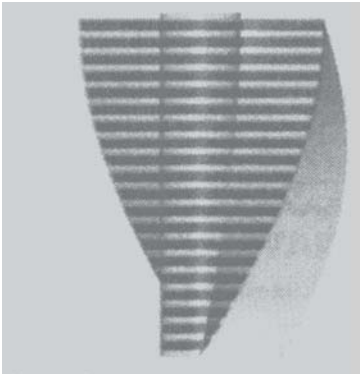
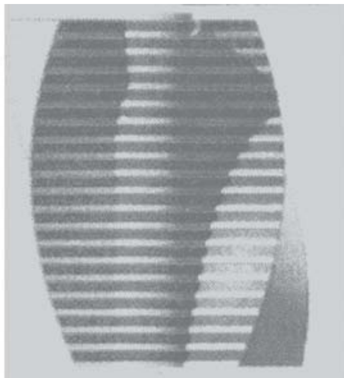
The following pictures are sideviews of the twisted building.



Sideview **1**



Sideview **2**

QUESTION 45.2

From which direction has Sideview 1 been drawn?

A. From the North.

B. From the West.

C. From the East.

D. From the South.

QUESTION 45.3

From which direction has Sideview 2 been drawn?

A. From the North West.

B. From the North East.

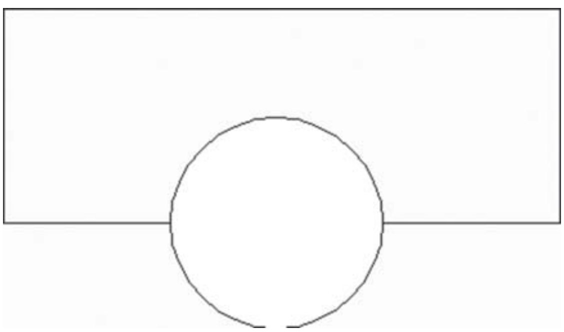
C. From the South West.

D. From the South East.

QUESTION 45.4

Each storey containing apartments has a certain 'twist' compared to the ground floor. The top floor (the 20th floor above the ground floor) is at right angles to the ground floor.

The drawing below represents the ground floor.



Draw in this diagram the plan of the 10th floor above the ground floor, showing how this floor is situated compared to the ground floor.

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MATHEMATICS UNIT 46: HEARTBEAT

For health reasons people should limit their efforts, for instance during sports, in order not to exceed a certain heartbeat frequency.

For years the relationship between a person's recommended maximum heart rate and the person's age was described by the following formula:

Recommended maximum heart rate = 220 - *age*

Recent research showed that this formula should be modified slightly. The new formula is as follows:

Recommended maximum heart rate = 208 – (0.7 x *age*)

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QUESTION 46.1

A newspaper article stated: "A result of using the new formula instead of the old one is that the recommended maximum number of heartbeats per minute for young people decreases slightly and for old people it increases slightly."

From which age onwards does the recommended maximum heart rate increase as a result of the introduction of the new formula? Show your work.

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QUESTION 46.2

The formula *recommended maximum heart rate* = 208 – (0.7 x *age*) is also used to determine when physical training is most effective. Research has shown that physical training is most effective when the heartbeat is at 80% of the recommended maximum heart rate.

Write down a formula for calculating the heart rate for most effective physical training, expressed in terms of age.

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MATHEMATICS UNIT 47 : SPACE FLIGHT

Space station Mir remained in orbit for 15 years and circled Earth some 86 500 times during its time in space.

The longest stay of one cosmonaut in the Mir was around 680 days.

QUESTION 47.1

Approximately how many times did this cosmonaut fly around Earth?

A. 110

B. 1 100

C. 11 000

D. 110 000

MATHEMATICS UNIT 48 : ROCK CONCERT

QUESTION 48.1

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A. 2 000

B. 5 000

C. 20 000

D. 50 000

E. 100 000

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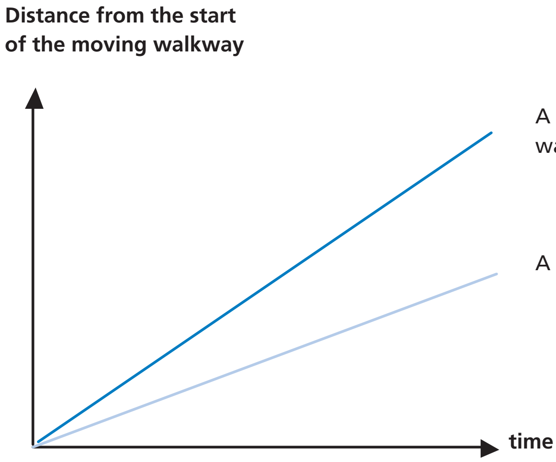
MATHEMATICS UNIT 49: MOVING WALKWAYS

QUESTION 49.1

On the right is a photograph of moving walkways.

The following Distance-Time graph shows a comparison between "walking on the moving walkway" and "walking on the ground next to the moving walkway."



Assuming that, in the above graph, the walking pace is about the same for both persons, add a line to the graph that would represent the distance versus time for a person who is standing still on the moving walkway.

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MATHEMATICS UNIT 50: POSTAL CHARGES

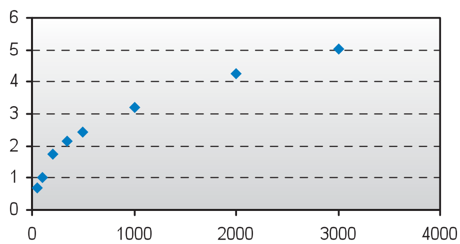
The postal charges in Zedland are based on the weight of the items (to the nearest gram), as shown in the table below:

Weight (to nearest gram)	Charge
Up to 20 g	0.46 zeds
21 g – 50 g	0.69 zeds
51 g – 100 g	1.02 zeds
101 g – 200 g	1.75 zeds
201 g – 350 g	2.13 zeds
351 g – 500 g	2.44 zeds
501 g – 1000 g	3.20 zeds
1001 g – 2000 g	4.27 zeds
2001 g – 3000 g	5.03 zeds

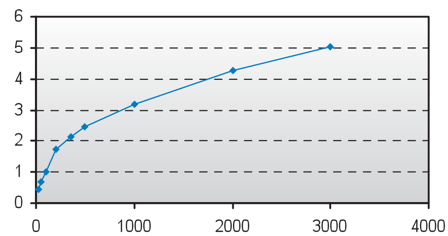
QUESTION 50.1

Which one of the following graphs is the best representation of the postal charges in Zedland? (The horizontal axis shows the weight in grams, and the vertical axis shows the charge in zeds.)

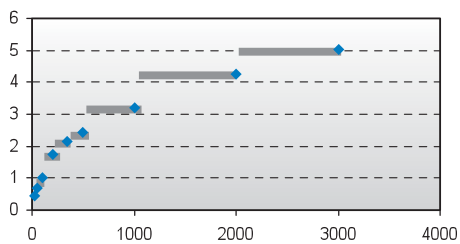
A



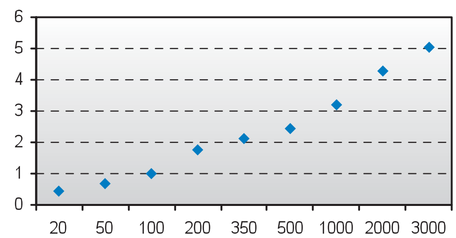
B



C



D



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QUESTION 50.2

Jan wants to send two items, weighing 40 grams and 80 grams respectively, to a friend.

According to the postal charges in Zedland, decide whether it is cheaper to send the two items as one parcel, or send the items as two separate parcels. Show your calculations of the cost in each case.

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